R-OSSE
Acoustic Waveguide

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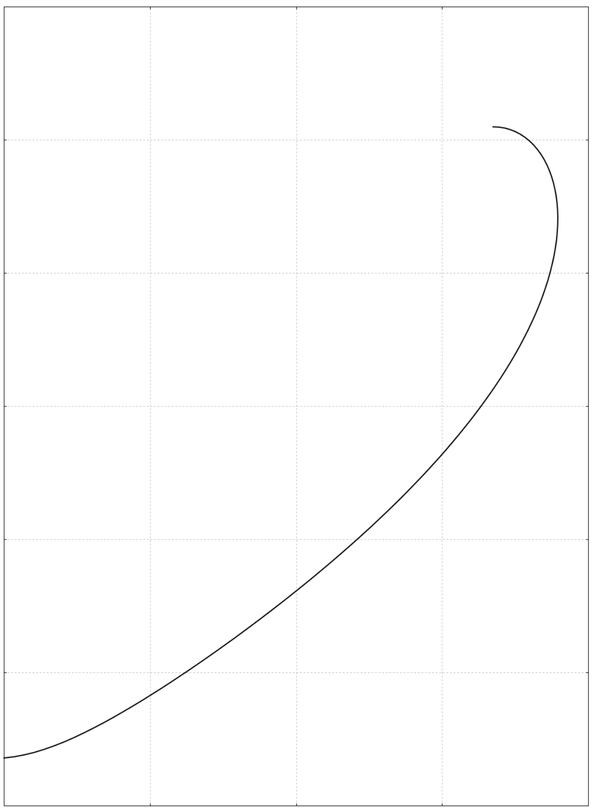


Introduction

In the previous work the OSSE ("OS-SE") waveguide formula was presented[1], extending the well-known Oblate Spheroidal (OS) waveguide by incorporating a smooth termination into a flat panel. While it proved the importance of the added gradual termination, due to its inherent half-space nature the usefulness was still somewhat limited - for a real-life use it is necessary to place such device into a finite baffle with an additional edge treatment which is not any more a part of its analytical description.

The now presented R-OSSE set of parametric equations goes a step further and defines a complete waveguide terminated into a free space by means of a convenient, self-containing analytical description. Such approach can be readily used e.g. in further optimization algorithms, CAD routines, etc.

The R-OSSE parametric description

In the following text we describe a profile of an axisymmetric waveguide as a set of coordinates [x, y], where 'x' is the axial distance from the throat and 'y' is the distance of the profile point from the axis.

Because the OSSE has the form of a function y(x), it can't describe shapes that fold back as the profile curve progresses (it can't have two different values for a single x). For this we need a parametric description in a form [x, y] = [x(t), y(t)], where x(t) and y(t) are some functions of a new parameter 't'. Typically these functions are constructed so that the parameter 't' ranges from 0 to 1.

The functions used in the R-OSSE description are plotted on the Fig 1.

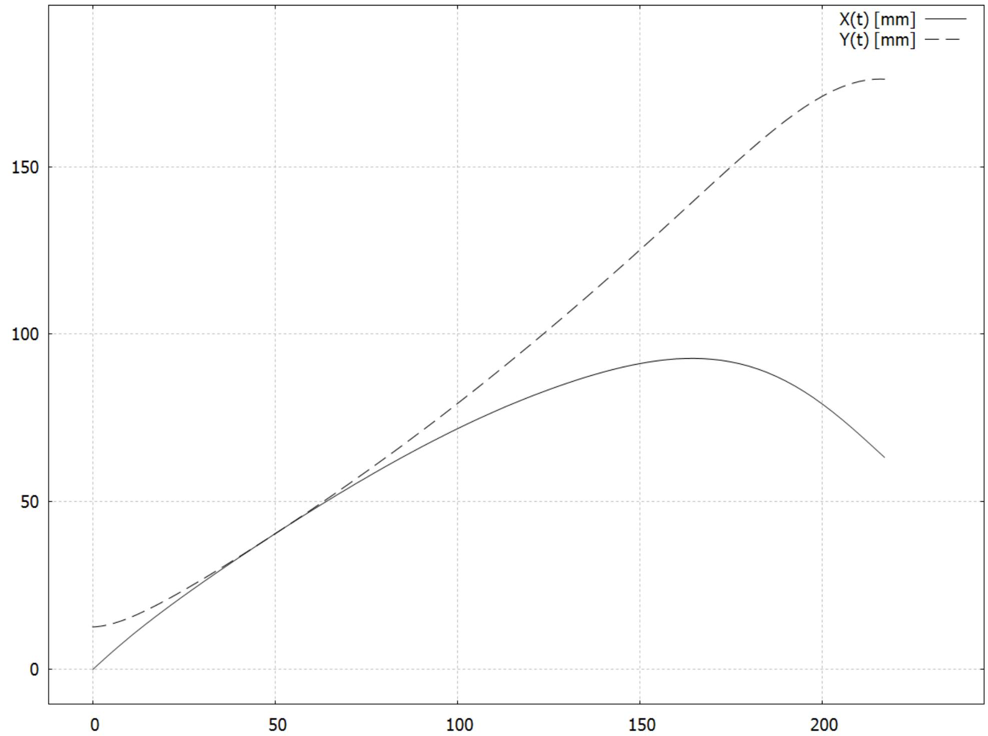


Fig. 1: R-OSSE x(t), y(t) components

1 http://www.at-horns.eu/release/OS-SE Waveguide.pdf

The functions on Fig. 1 are constructed by means of two conic sections each. The function x(t) is simply a difference of x1(t) and x2(t), a hyperbola and a parabola (Fig. 2). The function y(t) is a weighted average of y1(t) and y2(t), both being hyperbolas, starting as y2 and ending as y1 (Fig. 3).

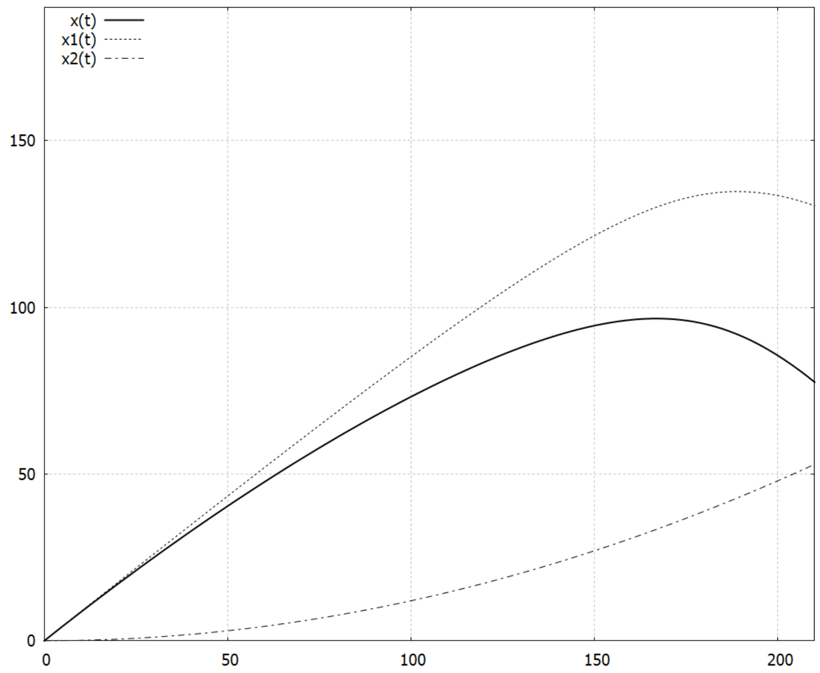


Fig. 2: x(t) components

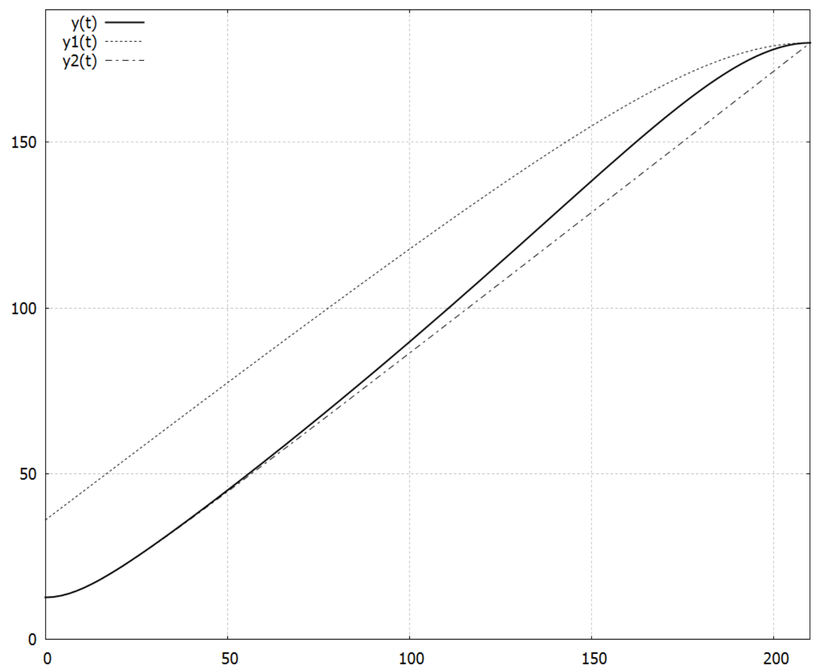


Fig. 3: y(t) components

R-OSSE design formulae

Design Parameter	**Description**	**unit/example**
R	Waveguide outer radius	[mm]/190
a	Nominal coverage angle	[deg]/40
r_0	Throat radius	[mm]/18
a_0	Throat opening angle	[deg]/0
k	Throat expansion factor	1
r	Apex radius factor	0.3
m	Apex shift factor	0.8
b	Bending factor	0.3
q	Throat shape factor	3

Auxiliary constants

$$k_1 = (kr_0)^2$$

$$k_2 = 2\,kr_0 \tan(a_0)$$

$$k_3 = \tan^2(a)$$

$$L = \frac{\sqrt{k_2^2 - 4k_3(k_1 - (R + r_0(k-1))^2)} - k_2}{2k_3}$$

Core functions

$$x_1(t) = L(\sqrt{r^2+m^2} - \sqrt{r^2+(t-m)^2})$$

$$x_2(t) = b\,x_1(1)\,t^2$$

$$y_1(t) = \sqrt{k_1 + k_2 L t + k_3 L^2 t^2} - r_0(k-1)$$

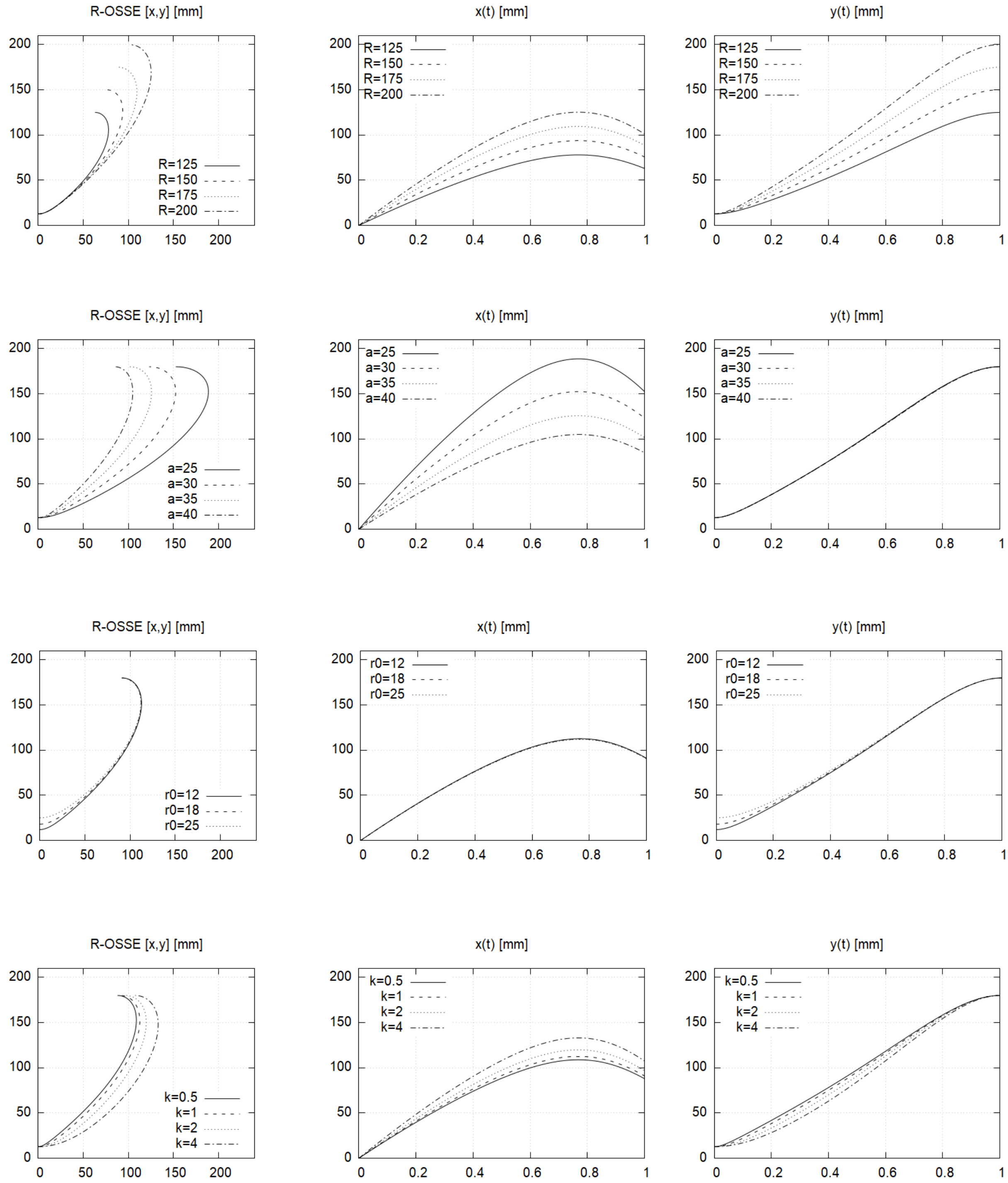
$$y_2(t) = R + L(1 - \sqrt{1 + k_3(t-1)^2})$$

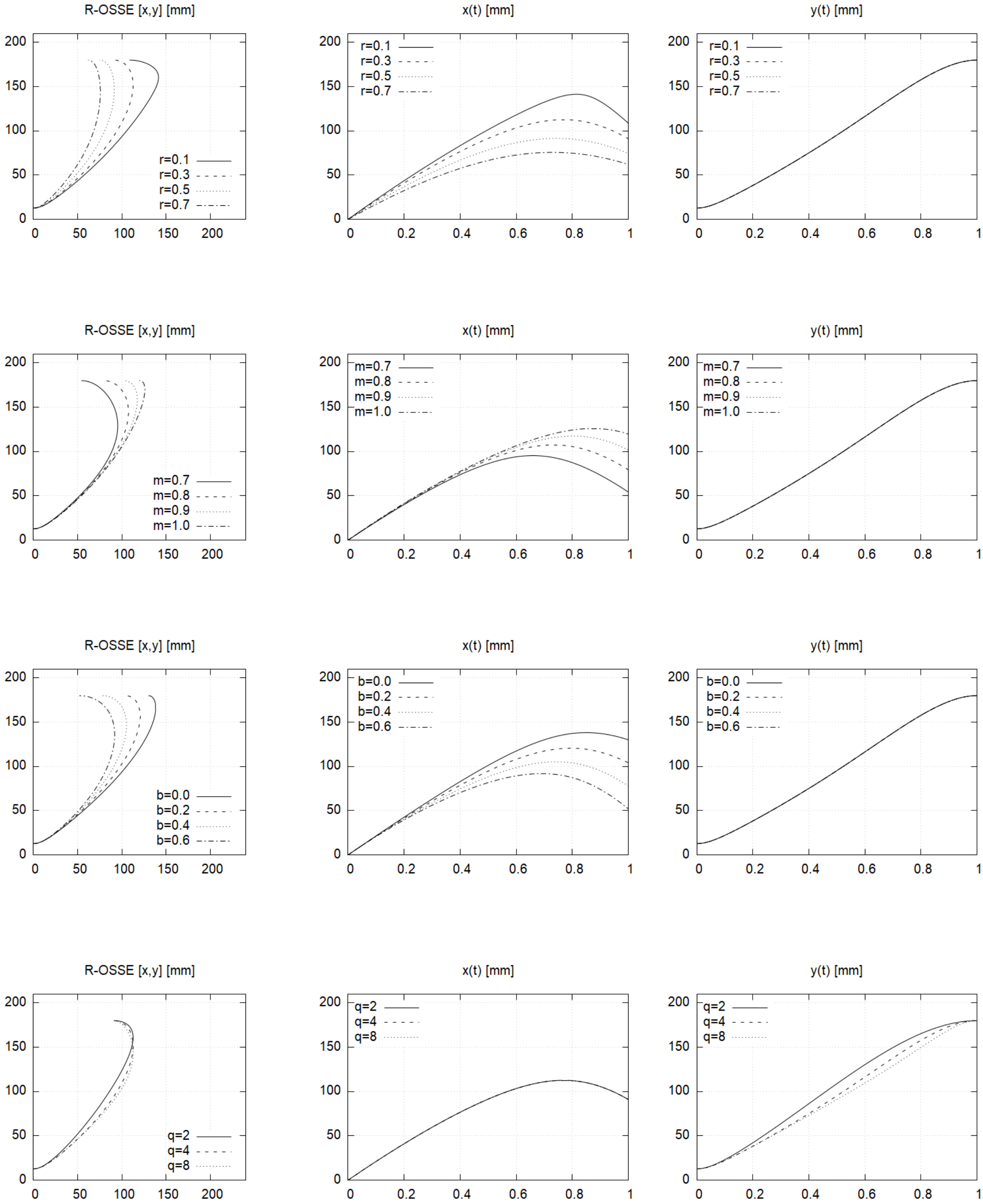
R-OSSE parametric equation

$$x(t) = x_1(t) - x_2(t)$$

$$y(t) = (1-t^q)\,y_1(t) + t^q\,y_2(t) \quad , \quad t \in \langle 0,1 \rangle$$

The following charts give an overview of the effect of each individual design parameter on the resulting shape.



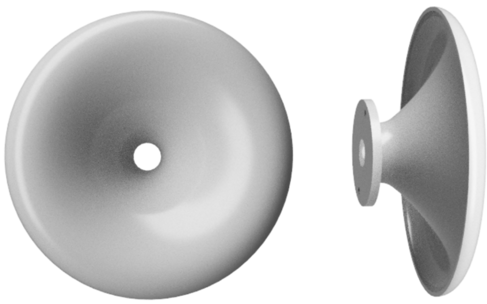

R-OSSE [x,y] [mm]
x(t) [mm]
y(t) [mm]
r=0.1
r=0.3
r=0.5
r=0.7
R-OSSE [x,y] [mm]
x(t) [mm]
y(t) [mm]
m=0.7
m=0.8
m=0.9
m=1.0
R-OSSE [x,y] [mm]
x(t) [mm]
y(t) [mm]
b=0.0
b=0.2
b=0.4
b=0.6
R-OSSE [x,y] [mm]
x(t) [mm]
y(t) [mm]
q=2
q=4
q=8

Practical design example

R-OSSE parameters

A sample waveguide[2] with 25.4 mm (1") throat is presented, given by the parameters listed below. This set of values results in a device that is 260 mm (~10") wide and slightly less than 80 mm (~3") long - see Fig. 4.

```
R = 130 mm          k = 1.8
r0 = 12.7 mm        r = 0.3
a0 = 7.5 deg        b = 0.3
a = 39 deg          m = 0.8
                    q = 3.7
```



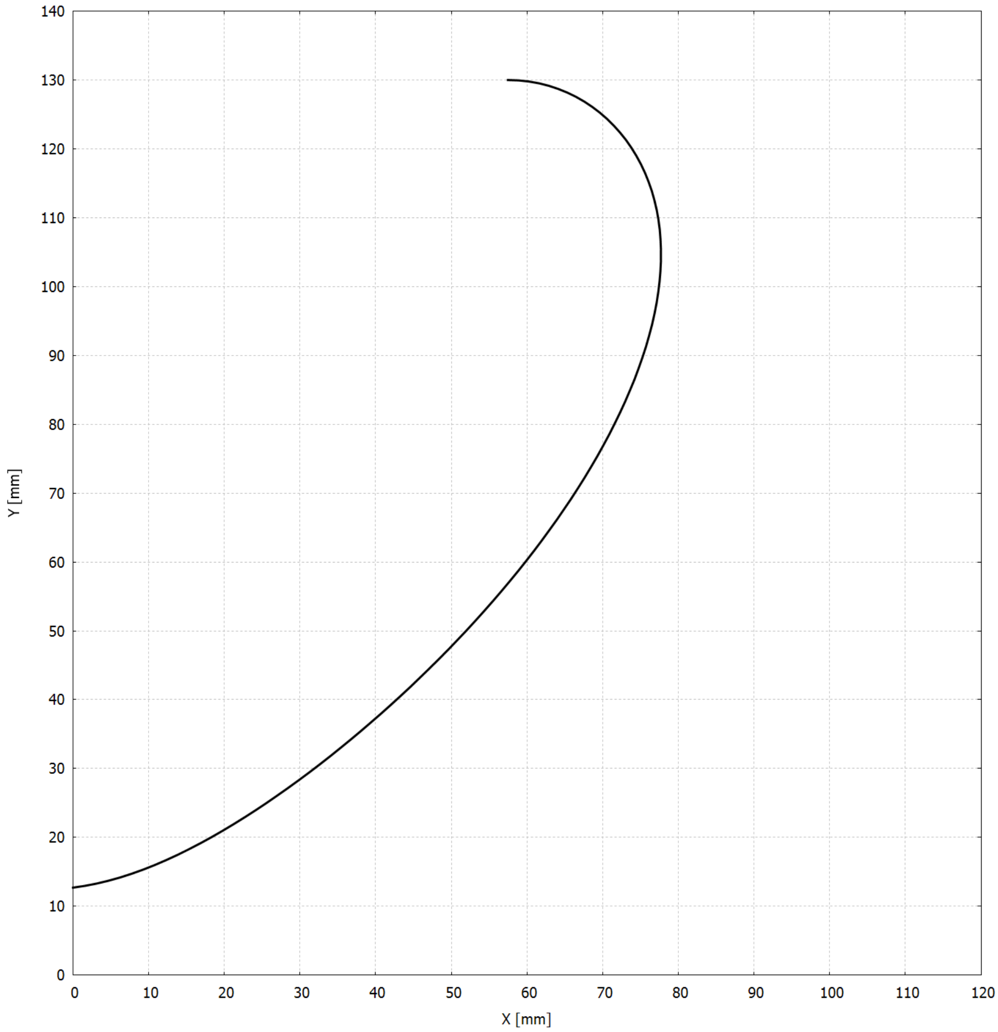


Fig. 4: Sample waveguide profile

2 This is an approximation of a device that has been around for some time, known as "ST260", made freely available for audio hobbyists, at the time constructed using a different and more complicated approach.

BEM simulation

The above axisymmetric device was numerically simulated in free field - as free standing with 5 mm thick wall - via a boundary element metohd (BEM), using software ABEC by R&D Team[3] (Mr. Joerg Panzer). 100 frequency points between 200 Hz and 20 kHz were used for the calculation. The results are presented in the following graphs.

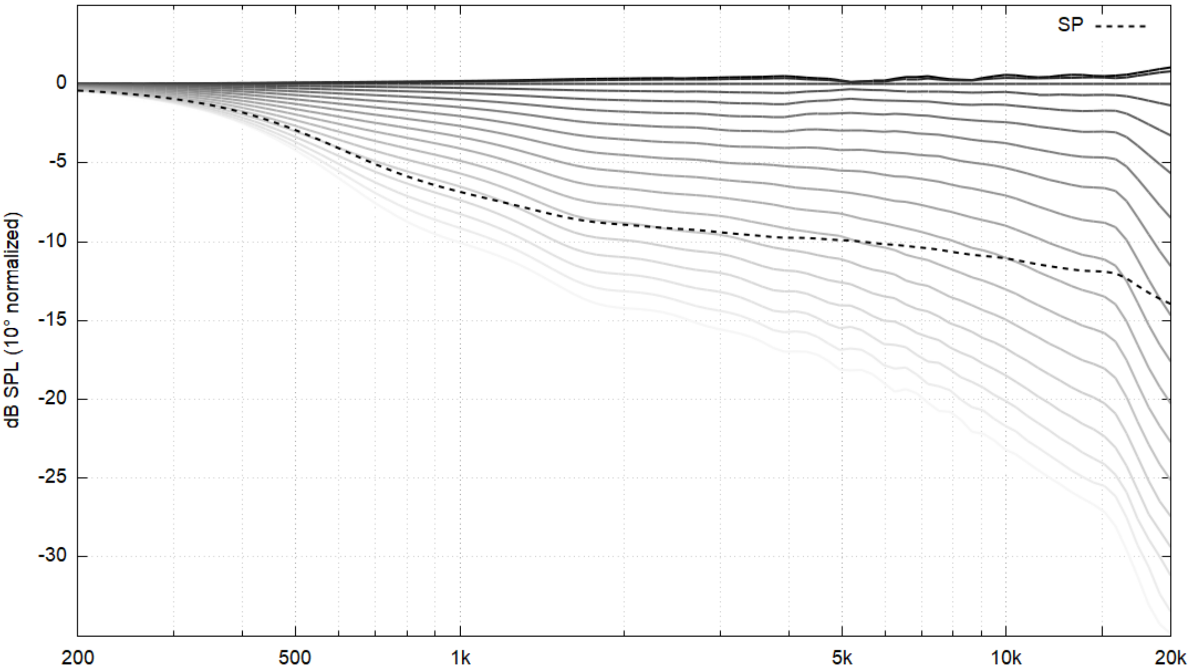


Fig. 5: BEM results - SPL polars, 0-90 deg / 5 deg step, 10 deg normalized

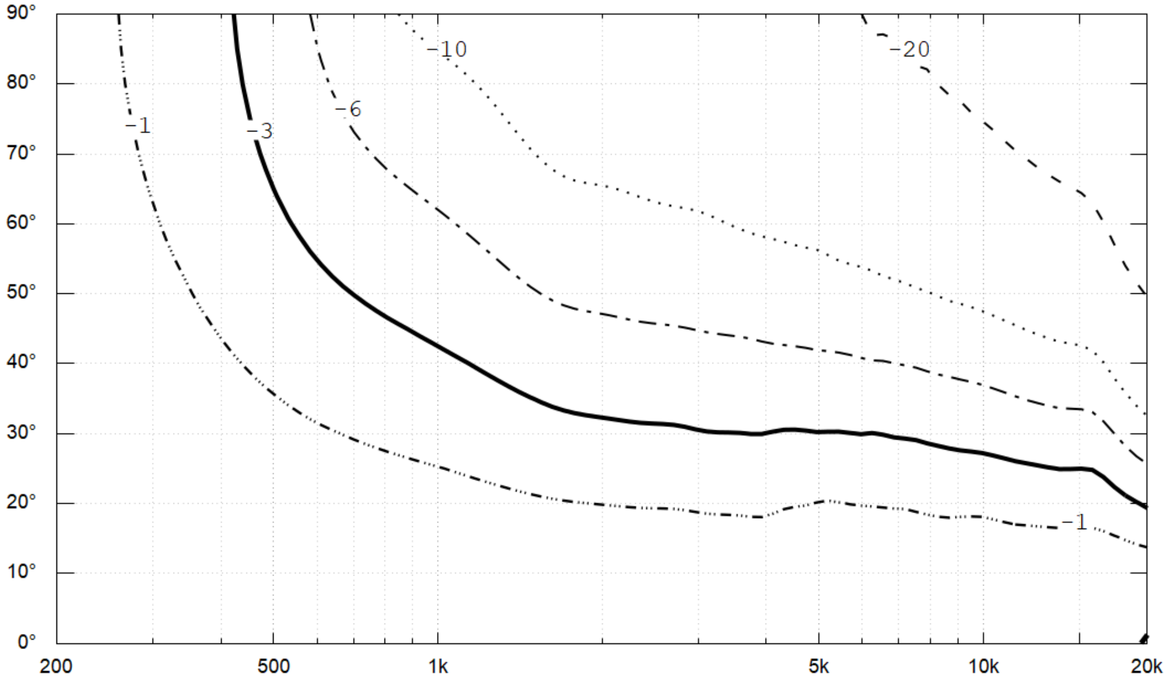


Fig. 6: BEM results - Polar map [dB SPL]

3 http://www.randteam.de

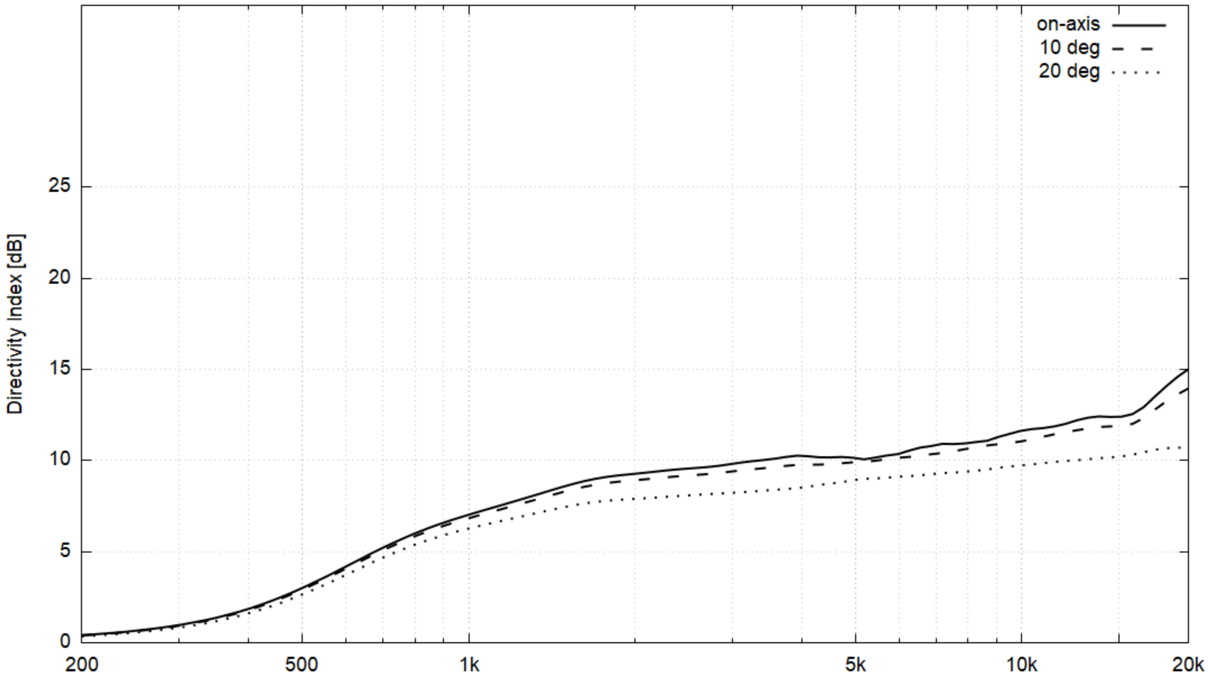


Fig. 7: BEM results - Directivity Index [dB]

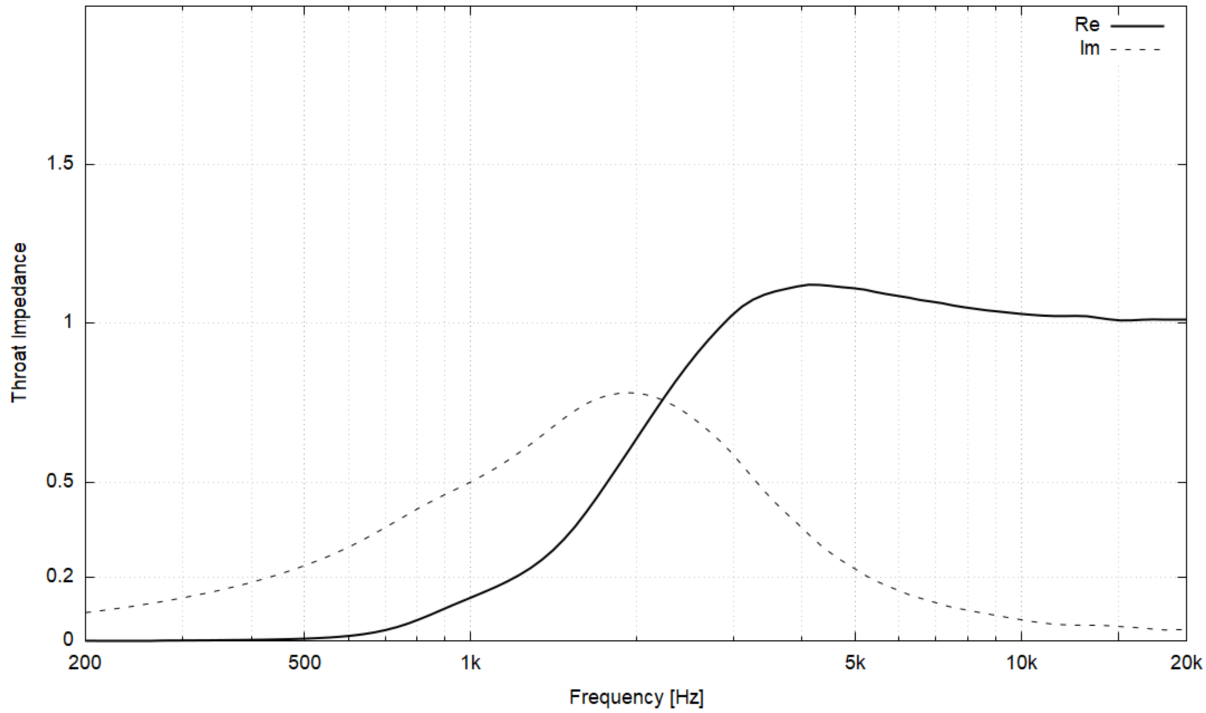


Fig. 8: BEM results - Throat acoustic impedance

Ath script code

For a reference, this is the Ath[4] script code that was used to create the above waveguide BEM mesh.

```
R-OSSE = {
  R = 130
  r0 = 12.7
  a0 = 7.5
  a = 39
  k = 1.8
  r = 0.3
  b = 0.3
  m = 0.8
  q = 3.7
}

Mesh.LengthSegments = 60
Mesh.AngularSegments = 8
Mesh.SubdomainSlices =
Mesh.WallThickness = 5

Source.Shape = 1

ABEC.SimType = 2
ABEC.SimProfile = 0
ABEC.MeshFrequency = 43000
ABEC.NumFrequencies = 100
ABEC.Abscissa = 1
ABEC.f1 = 200
ABEC.f2 = 20000

ABEC.Polars:SPL = {
  MapAngleRange = 0,180,37
  NormAngle = 10
}

Output.ABECProject = 1
Output.STL = 0

Report = {
  Title = ST260-ROSSE
  Width = 1600
  Height = 1000
}
```

4 Ath design tool - https://at-horns.eu